LECTURES ON REAL OPTIONS: PART IV — INFORMATION AND LEARNING

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Introduction

- So far, we have examined investment decisions for which all information comes from "nature":
  - Oil reserve: price of oil is determined in world market, and we simply observe it from day to day.
  - Price of widgets: goes up or down independently of what we do.
- Now we turn to investment decisions for which information comes from different sources:
  - Learning from our own actions. R&D, for example, yields information about the feasibility or cost of developing a new product.
  - Learning from others. A drug company might learn about the market potential for a new type of drug from the experience of its competitors.

Cost Uncertainty and Learning

- Actual cost of completing project is a random variable, $\tilde{K}$; only the expected cost $K = \mathcal{E}(\tilde{K})$ is known.
- Project takes time to complete — maximum rate at which firm can (productively) invest is k.
- On completion, firm receives asset (e.g., factory or new drug) whose value, V, is known with certainty.
- Expected cost K evolves according to

$$dK = -I\,dt + \nu\,(I\,K)^{1/2}\,dz \tag{1}$$

Cost Uncertainty and Learning (continued)

- All risk associated with dz is diversifiable.
- Note that K changes only if firm is investing, and variance of dK/K increases linearly with I/K.
- When firm is investing, expected change in K over Δt is $-I\ \Delta t$, but the realized change can be greater or less, and K can even increase.
- Variance of $\tilde{K}$ falls as K falls, but total cost of project, $\int_0^{\tilde{T}} I dt$, only known when project is completed.

Cost Uncertainty and Learning (continued)

- Problem: Find investment policy that maximizes value of investment opportunity, $F(K) = F(K; V, k)$:

$$F(K) = \max_{I(t)} \mathcal{E}_0 \left[Ve^{-\mu \tilde{T}} - \int_0^{\tilde{T}} I(t)e^{-\mu t}\, dt \right] \qquad (2)$$

subject to eq. (1), $0 \leq I(t) \leq k$, and $K(\tilde{T}) = 0$. Here μ is risk-adjusted discount rate, and time of completion, $\tilde{T}$, is stochastic.

Cost Uncertainty and Learning (continued)

- <u>Solution</u>: You can confirm that $F(K)$ must satisfy

$$\frac{1}{2}\nu^2 I\, K\, F''(K) - I\, F'(K) - I = r\, F(K)\,. \tag{3}$$

  - Eq. (3) is linear in I, so rate of investment that maximizes $F(K)$ is either zero or maximum rate k:

$$I = \begin{cases} k & \text{for } \frac{1}{2}\nu^2\, K\, F''(K) - F'(K) - 1 \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

  - There is a point K^*, such that $I(t) = k$ when $K \leq K^*$ and $I(t) = 0$ otherwise.

Cost Uncertainty and Learning (continued)

- K^* found with $F(K)$ by solving (3) subject to

$$F(0) = V\,, \tag{5}$$

$$\lim_{K\to\infty} F(K) = 0\,, \tag{6}$$

$$\tfrac{1}{2}\nu^2 K^* F''(K^*) - F'(K^*) - 1 = 0\,, \tag{7}$$

and condition that $F(K)$ is continuous at K^*.

Cost Uncertainty and Learning (continued)

- K can change only when $I > 0$, so if $K > K^*$ and firm is not investing, K will never change, and $F(K) = 0$.
- When $r = 0$, eq. (3) has analytical solution:

$$F(K) = V - K + \nu^2 \left(\frac{V}{2}\right)^{-2/\nu^2} \left(\frac{K}{\nu^2 + 2}\right)^{(\nu^2+2)/\nu^2} , \quad (8)$$

and critical value K^* is

$$K^* = \left(1 + \tfrac{1}{2}\nu^2\right) V.$$

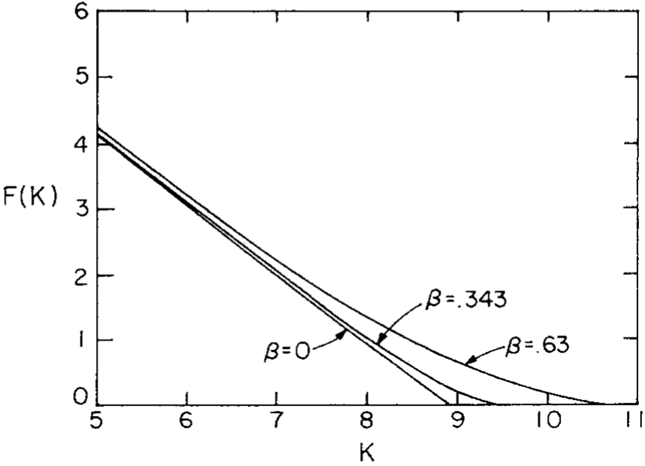
Cost Uncertainty and Learning (continued)

- Eq. (8) has simple interpretation. When $r = 0$, $V - K$ would be value of investment opportunity were there no possibility of abandoning the project. The last term is the value of put option, i.e., option to abandon should cost turn out to be higher than expected.
- Note that for $\nu > 0$, $K^* > V$, and K^* is increasing in ν. The more uncertainty there is, the greater the value of the investment opportunity, and the larger is the maximum expected cost for which beginning to invest is economical.
- Here, simple NPV might say don't invest, but in fact you should invest.
- When $r > 0$, use numerical solution. For $r = .05$, $V = 10$, and $k = 2$, table shows K^* and $F(K^*)$ for different values of ν.

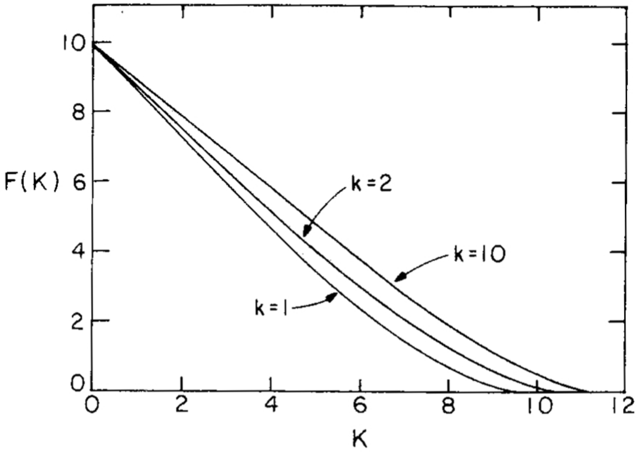
Cost Uncertainty and Learning (continued)

ν	K^*	$F(K^*)$
0	8.9257	0
0.1	8.9844	0.1384
0.2	9.1309	0.2026
0.3	9.3750	0.2428
0.4	9.7168	0.3924
0.5	10.156	0.5199
0.6	10.693	0.7499
0.7	11.328	0.9067
0.8	12.051	1.1664
0.9	12.861	1.3606
1.0	13.770	1.6034

Technical Uncertainty



Changes in Maximum Rate of Investment



Cost Uncertainty and Learning (continued)

- Figures show $F(K)$ as a function of K for three values of ν, and for different values of k.
- How to estimate ν in practice? Make use of the fact that variance of cost to completion is

$$\mathcal{V}(\tilde{K}) = \left(\frac{\nu^2}{2 - \nu^2} \right) K^2. \tag{9}$$

(Thus if one S.D. of a project's cost is 25% (50%) of the expected cost, ν would be 0.343 (0.63).) Using (9) and an initial estimate of expected cost, $K(0)$, a value for ν can be based on an estimate of the S.D. of $\tilde{K}$.

Generalization of Model

- In Dixit & Pindyck, Chap. 10, model also includes input cost uncertainty:

$$dK = -Idt + \nu(IK)^{1/2}dz + \gamma K dw$$

  where dz and dw are increments of uncorrelated Wiener processes, and dw may be partly non-diversifiable.

- For nuclear power plant construction and abandonment decisions during the 1980s, $\gamma K dw$ term is crucial. See R. Pindyck, "Investments of Uncertain Cost," in *Journal of Financial Economics*, 1993.

Generalization of Model(continued)

- Schwartz and Moon generalized model to also include
  - Asset value uncertainty. V evolves as

$$dV = \mu V dt + \sigma V dw$$

  - Catastrophic events. Possible arrival (Poisson process) of "catastrophic event" that drives V to zero.
  - They use numerical solution method, and apply model to drug development and FDA testing.

Schwartz-Moon Model

- Cost Uncertainty:

$$dK = -Idt + \beta(IK)^{1/2}dz$$

where $K = \mathcal{E}(\tilde{K})$, and $\tilde{K}$ is actual cost to completion.

  - Var $(\tilde{K}) = \left(\frac{\beta^2}{2-\beta^2}\right) K^2$
  - $I_m = k =$ maximum rate of investment.

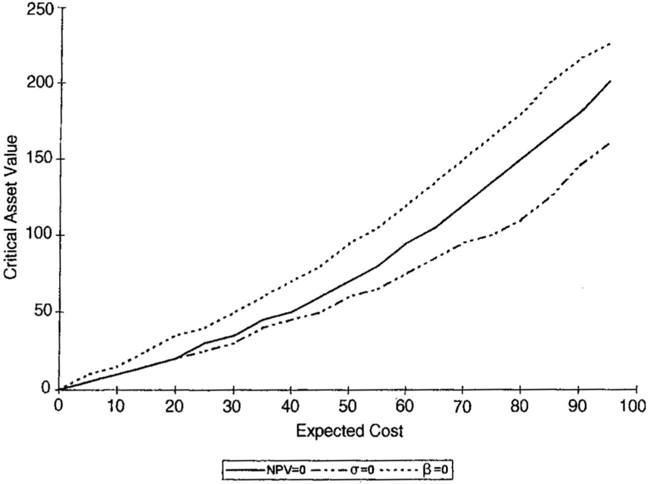
- Asset Value Uncertainty:

$$dV = \mu V dt + \sigma V dw$$

- "Catastrophic Event": λ is probability (per year) that V jumps to 0.

- Investment Rule: Invest at maximum rate when $V \geq V^*(K)$; $I = 0$ when $V < V^*(K)$.

Critical Asset Values for Different Expected Completion Costs



Application to Development of New Drug

- Extend model to 4 distinct phases: Phase I testing, Phase II testing, Phase III testing, FDA review.
  - Each phase has different characteristics: amount of investment, maximum rate of investment, and probability of failure or catastrophic event during the phase.
  - There may be differences in uncertainty related to cost to completion and to asset value obtained at completion.
- Like a compound option: Completion of first phase gives option to start second phase, and so on.
  - Boundary condition of each phase is the beginning value for the next phase.
  - Problem is solved recursively starting from last phase, which has as its terminal boundary the asset value at completion.
  - Penultimate phase has as its terminal boundary the value of the project before last investment phase is done, and so on.

Application to Development of New Drug

- Table 1 shows data.

Table 1: Data for New Drug Development

	Phase I	*Phase II*	*Phase III*	*FDA review*
Expected cost (million)	4	10	60	10
Maximum rate of investment/year (million)	2	5	20	4
Probability of failure (λ)	0.15	0.25	0.06	0.01
Asset return volatility (σ)	0.35	0.35	0.35	0.35
Drift of asset value process (μ)	0.0	0.0	0.0	0.0
Volatility of cost (β)	0.50	0.50	0.50	0.50
Interest rate (r)	0.50	0.05	0.05	0.05
Risk premium (η)	0.08	0.08	0.08	0.08

Application to Development of New Drug

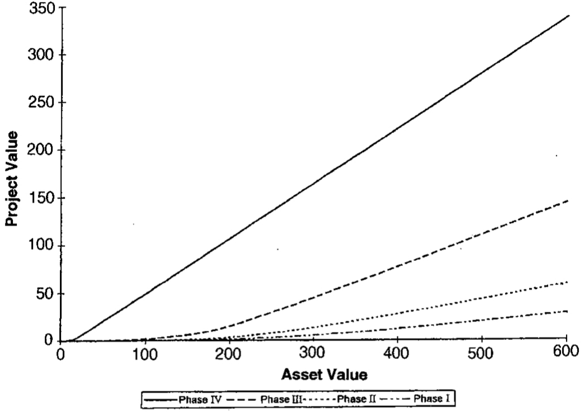
- Table 2 shows V^* at beginning of each stage.

Table 2: Evaluation of New Drug Development

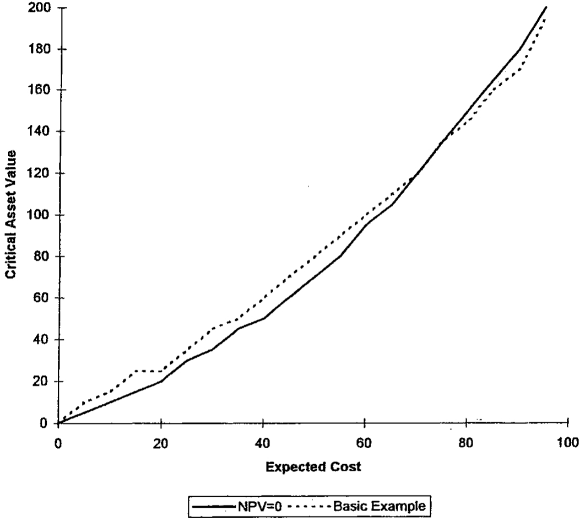
	For asset value V	*Phase I*	*Phase II*	*Phase III*	*FDA review*
Critical asset value (V^*)		250	215	180	15
Project value	300	5.7	13.9	45.2	164.7
Project volatility	300	1.15	1.15	0.98	0.43
Project beta	300	3.06	2.82	2.13	1.05
Project value	500	20.2	43.3	110.9	280.0
Project volatility	500	0.78	0.76	0.64	0.42
Project beta	500	2.00	1.80	1.50	1.03

Project Value At Each of Four Phases of New Drug Development

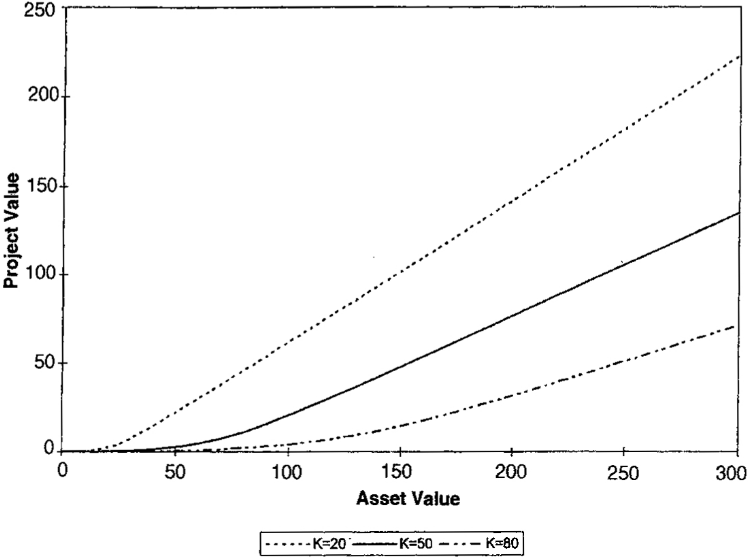
- Figure shows project value $F(V, K)$, as a function of V, at beginning of each stage.



Critical Asset Values vs. Expected Completion Costs: Basic Example and $NPV = 0$



Project Value vs. Asset Value ($K = 20, 50, 80$).



Information and Strategic Options

Learning from others:

- Return to simple widget factory example we looked at earlier. But now, 2 firms:
- Price = \$50 per widget, or \$150, with equal probability. Cannot find out until you—or the other firm— actually invests.
- Each firm can invest at cost of \$800. Hence, for each firm, NPV of investing now is:

$$\mathrm{NPV}_i^{\mathrm{NOW}} = -800 + \sum_{t=0}^{\infty} 100/(1+R)^t \tag{10}$$

$R = .10$, so $\mathrm{NPV}_i^{\mathrm{NOW}} = -800 + 1100 = \$300$.

Information and Strategic Options (continued)

- Suppose Firm 2 will invest now. Should Firm 1 wait? If it waits, it will only invest if it learns that $P = \$150$. NPV for Firm 1 from waiting is then:

$$\mathrm{NPV}_1^{\mathrm{WAIT}} = \tfrac{1}{2}\left[-\frac{800}{1.1} + \sum_{t=1}^{\infty} 150/(1.1)^t\right] = \$386 \qquad (11)$$

- Hence, better to wait. But Firm 2 is thinking the same thing. Suppose neither firm invests now. If, at end of year, both firms invest (without benefit of knowledge), NPV today will be:

$$\mathrm{NPV}_i^{\mathrm{WAIT}} = 300/1.1 = \$273 \qquad (12)$$

Information and Strategic Options (continued)

We thus have a gaming situation:

		Firm 2: Invest Now	Firm 2: Wait
Firm 1	Invest Now	300, 300	300, 386
	Wait	386, 300	273, 273

Information and Strategic Options (continued)

- How would you play this game if you were Firm 1? Might assign a probability p that Firm 2 will invest now. Then,

$$\mathrm{NPV}_1^{\mathrm{NOW}} = 300$$

$$\mathrm{NPV}_1^{\mathrm{WAIT}} = (p)(386) + (1-p)(273) = 273 + 113p$$

  Hence it is better to wait if $273 + 113p > 300$, or $p > .24$

- Of course there is no reason for this process to stop at the end of one year. In this war of attrition, it is possible for a very long time to go by with neither firm investing.

Informational Cascades

- People are deciding whether to buy real estate in downtown Oskosh.
- If a person acts, ultimate payoff will be the fundamental value V, which is either 1 or -1, intitially with probability $1/2$.
- Individuals receive *signals* — either *high* (H) or *low* (L).
  - If $V = 1$, signal will be H with probability $p > \frac{1}{2}$ and L with probability $(1 - p) < \frac{1}{2}$.
  - If $V = -1$, signal will be H with probability $(1 - p)$ and L with probability p.
  - So, a signal is informative, but does not eliminate all uncertainty.

Informational Cascades (continued)

- **Observable Signals.** Suppose new signal comes every week, observed by all potential investors. What will happen?
  - As number of signals increases, uncertainty over true V is reduced.
  - Eventually all investors settle on correct choice: They invest if $V = 1$ and don't if $V = -1$.
- **Observable Actions.** Suppose each person receives one signal, and you can only observe the *actions* of others.
- Can lead to informational cascade in which many people invest even though in fact $V = -1$, or many people don't invest even though $V = 1$.
- Consider sequence of risk-neutral investors, A, B, C, etc. Want to know what each will do given his own signal, and given observed *actions* of predecessors.

Informational Cascades (continued)

- A will invest if his signal is H, will not if signal is L. Note all others can infer A's signal from his action.
- Suppose A invested. What should B do?
  - Clearly B should invest if her signal is H.
  - If B's signal is L, her posterior probability that $V = 1$ is $\frac{1}{2}$, so she is indifferent.
  - Assume in this case she flips a coin.
- C faces three possibilities:
  1. A and B both invested. Then C will invest, even if his signal is L. (NPV is positive, no matter what signal C received, and even though B may have flipped a coin.)
  2. Neither A nor B invested. Then C will not invest, even if his signal is H.
  3. A invested and B did not, or vice versa. Then C will only invest if his signal is H.

Informational Cascades (continued)

- Let's focus on first case: A and B both invested, and thus C invested even if his signal is L. Now, what will D do?
- D will invest, no matter what signal she receives.
- Likewise, E, F, etc. will all invest.
- We now have an informational cascade. It is possible that A, *and only A*, received a signal of H, and all others received signals of L. Yet all will invest.
- If D, E, etc., could have observed that B and C received signals of L, they would not have invested. But they only observe *actions* of others.
- Everyone is acting rationally (expected NPVs are positive) even though *no new information is being produced.*

Informational Cascades (continued)

- Suppose in fact $V = -1$. Then how does this end? Perhaps some investors seek and obtain information – signals of L. They start to sell. Others observe these actions and — quite rationally — also sell. Price plummets!
- **What to Do?** Doesn't mean you should not invest. (Expected NPVs are positive.) But understand how little information your decision is based on.
- <u>Remember</u>: Rational decisions based on *actions* of others involve much more risk than decisions based on accumulation of fundamental information.